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Composite correlations for convective heat transfer from arrays of three-dimensional obstacles

GARRON K. MORRIS and SURESH V. GARIMELLA†

Department of Mechanical Engineering, University of Wisconsin-Milwaukee, P.O. Box 784, Milwaukee, WI 53201, U.S.A.

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INTRODUCTION

The objective of the present work is to develop empirical correlations applicable for single-phase forced-convection heat transfer from inline arrays of three-dimensional heated elements in channels. The correlations should be applicable to the cooling of miniaturized electronic components and computer chips, turbine-blade internal flow passages, and other heat transfer enhancement situations. In an effort to generalize the results from particular sets of experiments to a range of geometric, thermal, and flow configurations, thermal wake functions (which use the principle of superposition) have been proposed or used to predict the local temperatures in an array of heated protruding elements in air flow [1–5]. Correlations restricted to air [4, 5] and liquid flows [6] have also been proposed. Although temperatures predicted by these methods have compared favorably with experimental data, there is a continuing need for composite correlations that can predict element-averaged heat transfer coefficients in air and liquid flows.

The proposed correlations are based on data from 633 experiments that cover a wide range of flow rates (chip length-based Reynolds numbers from 600 to 70 000), geometries, array spacings and Prandtl numbers (0.7–25.2). The data include those from in-house (unpublished) experiments as well as from a number of studies in the literature [1, 4, 5–9]. Air, water, and FC-77 are the coolants included. There is a host of other studies in the literature dealing with flow past wall-mounted obstacles such as single elements and two-dimensional (2D) ribs. However, only data for inline arrays of 3D elements are included in the present effort.

THE EXPERIMENTAL DATABASE

All the experiments in the database were performed in horizontal channels of uniform height H and width W. A fluid with a mean upstream velocity u_m was forced over an array of elements ($L \times L_z$ in plan and B high) with streamwise and spanwise spacings between elements of LS and SS, respectively. Figure 1 illustrates a typical array configuration. A summary of the experimental parameters for each study can be found in Table 1.

In-house experiments

These experiments [9] were performed on the same experimental facility as described in Garimella and Schlitz [6]. The experiments were conducted using degassed distilled water in a closed-loop flow facility at different inlet temperatures and flow rates. The horizontal test section was 60 cm long and 11 cm wide. The thickness of the removable top wall was varied to yield three different channel heights of 0.1905, 2.0 and 4.0 cm. Several screens of varying mesh size and a section of honeycomb installed upstream of the test section ensured a uniform inlet flow field.

Arrays of 15 rows and five columns of elements were mounted on the bottom wall of the test section with a fixed streamwise (LS) and spanwise (SS) spacing of 0.61 cm. The single heated element was located in the tenth row of the central column; the remaining elements in the array were unheated. The elements used measured 1.016 cm on the side (L) with a height (B) of either 0.0635 or 0.432 cm. The heat source was constructed of copper and heated with a cartridge heater installed into a copper stem underneath the element. The insulation scheme and the means of determination of the actual heat output from the exposed part of the element are described in Garimella and Schlitz [6]. The experiments were conducted at a nominal power output of 11 W. The inlet flow was maintained at constant temperatures of 20, 30 and 40°C, with corresponding Prandtl numbers of 6.97, 5.42 and 4.34, respectively. The experiments were performed at element length-based Reynolds numbers ($u_m L/\nu$) ranging from 610 to 68 580.

The adiabatic heat transfer coefficient was calculated according to:

$$\overline{h_{ad}} = q''_h/(T_h - T_i) \tag{1}$$

where q''_h is the heat flux through the exposed area of the heated element, T_h is the temperature of the heated element, and T_i is the temperature of the inlet flow. Uncertainties in the heat transfer coefficients were reported to be less than $\pm 4\%$ at a 95% confidence level.

Experiments in the literature

Experiments from a number of studies in the literature [1, 4, 5–8] were included in the database. A summary of the experimental parameters is presented in Table 1. The results obtained in all the experiments considered in this paper represent "adiabatic" heat transfer coefficients; the significance of this concept was first discussed by Moffat and co-workers [2]. The temperature of an element in a heated array was termed adiabatic when power to that particular element was turned off (thus representing the heating due to the thermal wakes from the upstream elements alone). This temperature is used as the reference to calculate adiabatic heat transfer coefficients. The heat transfer coefficient for a single heated element in an array of unheated elements is also, by defi-

† Author to whom correspondence should be addressed.

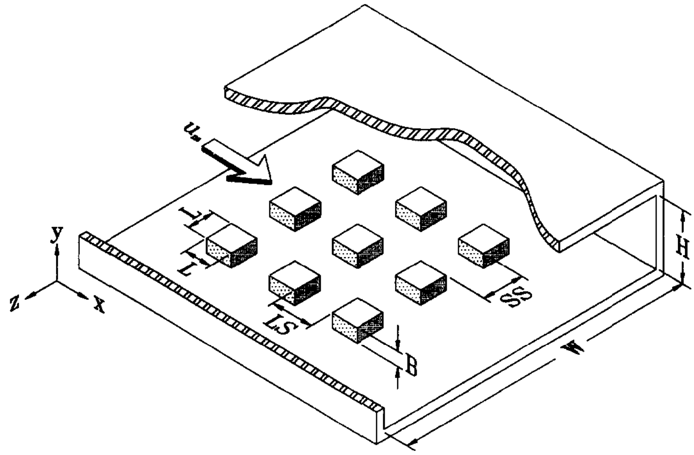

Fig. 1. Typical channel and array configuration.

nition, adiabatic in this sense. In this case, the heat transfer coefficient may be calculated based on the adiabatic element temperature, the inlet fluid temperature, or the bulk-mean fluid temperature, all of which are identical.

THE CORRELATIONS

The development of the correlation parameters is first presented. A correlation is proposed next for all the experiments listed in Table 1; a separate correlation is then presented for the air data alone. Finally, the proposed correlations are compared with predictive formulae in the literature.

Development of the correlation parameters

As established by Garimella and Eibeck [7], the flow becomes hydrodynamically and thermally fully developed by the fourth row of an array of elements. Thus, the row-independent, adiabatic heat transfer coefficient is a function only of the flow rate, fluid properties, channel and element geometries, and array spacing. In the literature, various characteristic lengths have been used to define the Reynolds number including the element length or height, channel height, channel hydraulic diameter, and clearance $(H-B)$. The Nusselt number is typically characterized by either the element length or height. The temperature at which the thermophysical properties are evaluated becomes important when the fluid properties vary strongly with temperature (as in the case of FC-77) and where large temperature differences are encountered. The streamwise length of the element was found to be the optimal choice for the characteristic length in the definition of the Reynolds $(u_m L/\nu)$ and Nusselt numbers $(\overline{h}_{ad} L/k)$, with the fluid properties evaluated at the inlet temperature, T_i.

The Nusselt numbers for all the experiments included in this paper are plotted as a function of Reynolds number in Fig. 2. As expected, the magnitude of Nusselt number increases linearly with Reynolds number in the log–log plane. The influence of Prandtl number $(c_p\mu/k)$ is evident in the groupings of the air, water, and FC-77 data. Although the magnitudes of the heat transfer coefficients differ in these groupings, they appear to have similar slopes.

Of the various dimensionless geometric-parameter definitions, the streamwise array spacing-to-channel height ratio (LS/H) and channel clearance-to-element length ratio $[(H-B)/L]$ were found to best account for the channel and array geometry effects on heat transfer. The relation between the dimensionless parameters can thus be stated as:

$$Nu_L = aRe_L^b Pr^c (\mu_i/\mu_h)^d (LS/H)^e [(H-B)/L]^f \quad (2)$$

The properties in Nu_L, Re_L and Pr are all evaluated at the inlet temperature. The fluid viscosity is the property most affected by temperature, and this variation is accounted for in the viscosity ratio [10].

Overall heat transfer correlation

The heat transfer data from the experiments presented in Table 1 were correlated with nonlinear least-squares curve fits using an adaptive combination of the Gauss–Newton and Levenberg–Marquardt algorithms. In the curve fits, the exponent of the Prandtl number is indicated to be much larger than the usual value of around 0.33. It was therefore fixed at 0.5 in the correlations of this paper, as also done in Garimella and Schlitz [6]. Lower exponents for Prandtl number resulted in significant deterioration of the predictions. There is evidence in the literature [11] that the Prandtl number exponent is expected to be higher when roughened surfaces are used (as in the present study), compared to the exponents of 0.33–0.4 for smooth surfaces.

The proposed correlation is:

$$Nu_L = 0.158 Re_L^{0.655} Pr^{0.5} (\mu_i/\mu_h)^{0.13} \times (LS/H)^{0.125} [(H-B)/L]^{0.05} \quad (3)$$

which is valid for: $610 < Re_L < 68\,580$; $0.71 < Pr < 25.2$; $0.025 < LS/H < 3.22$ and $0.06 < [(H-B)/L] < 3.87$. The Reynolds number exponent (0.655) compares favourably with Reynolds number dependencies in the literature of 0.718 [4], 0.60 [5] and 0.663 [6]. Garimella and Eibeck [7] reported that the streamwise spacing between elements significantly influenced heat transfer and included LS as a parameter in their Nusselt number correlation. Their (LS/B) exponent was 0.15, compared to the (LS/H) exponent of 0.125 proposed in the present correlation. Lehmann and Pembroke [4] found the Nusselt number to be essentially independent of $[(H-B)/L]$ when the element length L was used as the characteristic length in Reynolds number. The present low exponent of 0.05 for $[(H-B)/L]$ is consistent with their findings.

Considering both liquid and air data (633 experiments), predictions using equation (3) differ from the experiments by an average error of $\pm 9.4\%$ and a standard deviation of 10%. The liquid (water and FC-77) experimental data are predicted by equation (3) to a lower average error ($\pm 6.8\%$) and standard deviation (5.2%). The experimental data are plotted along with predictions from equation (3) in Fig. 3(a). The majority of the experiments are well correlated; however, scatter in the air data is evident. It appears that the data of Anderson and Moffat [1] and Rhee *et al.* [8] show a Nusselt number dependence on LS/H that is larger

Table 1. Experiments used in the present study

	Fluid	Re_L	H [cm]	W [cm]	L [cm]	L_z [cm]	B [cm]	LS [cm]	SS [cm]	LS/H	$(H-B)/L$
Anderson and Moffat [1]	Air	7430–20 253	2.14 2.85 4.37	35.6	3.75	4.65	0.95	1.27	1.27	0.29 0.45 0.59	0.32 0.51 0.91
Garimella and Eibeck [7]	Water	624–9041	1.4, 2.2 3.2, 4.2	36.6	2.54	2.54	1.167	0.58, 1.28 2.57, 5.02 7.59	0.58 2.57 7.59	†	0.092, 0.41 0.80, 1.19
Garimella and Schiltz [6]	FC-77	762–40 000	0.9 2.0	11.0	1.016	1.016	0.0635 1.016 1.504	0.61	0.61	0.31 3.20	0.13 0.49 0.97
Gudapati [9]	Water	610–68 580	0.19 2.0 4.0	11.0	1.016	1.016	0.0635 0.432	0.61	0.61	0.15 0.31 3.20	0.13, 1.54 1.91, 3.51 3.87
Lehmann and Pembroke [4]	Air	1500–31 250	1.92 3.56 5.65	15.2	2.743	2.743	0.159	0.343	0.343	0.061 0.096 0.18	0.64 1.24 2.0
Rhee *et al.* [8]	Air	2235–9591	6.75 12.0	30.5	3.0	3.0	3.0	0.3 3.0 7.0	0.3 3.0 7.0	0.025, 0.044 0.25, 0.44 0.58, 1.04	1.25 3.0
Wirtz and Dykshoorn [5]	Air	1267–13 000	0.79, 0.95 1.27, 1.91 2.93	25.4	2.54	2.54	0.635	2.54	2.54	0.87, 1.33 2.0, 2.67 3.22	0.061, 0.13 0.25, 0.50 0.91

† LS/H = 0.14, 0.27, 0.31, 0.40, 0.42, 0.61, 0.80, 0.92, 1.17, 1.19, 1.81, 1.83, 2.28, 2.37, 3.58, 5.42.

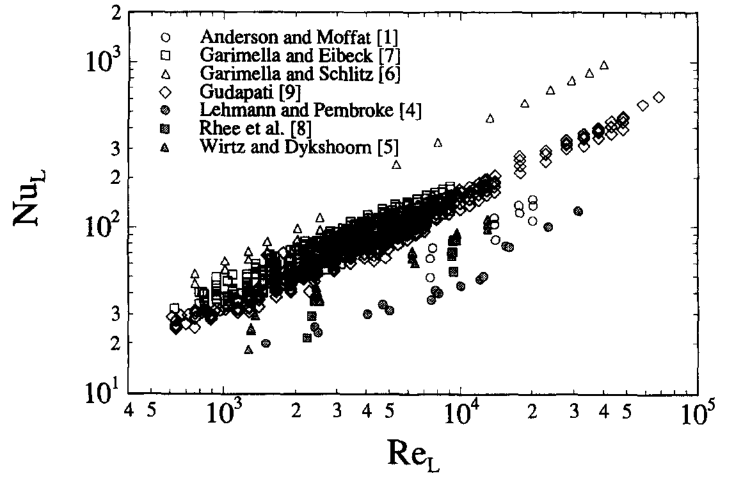

Fig. 2. Nusselt numbers for the air, water, and FC-77 data as a function of Reynolds number.

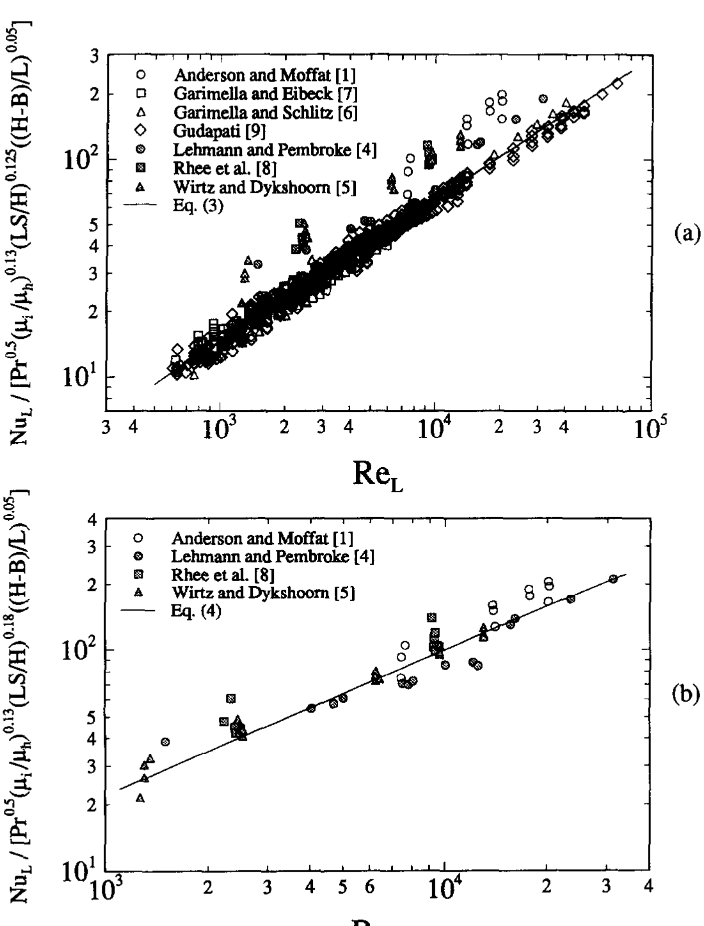

Fig. 3. Comparison of the experimental data with equations (3) and (4).

(exponent = 0.184) than used in the present correlation. This may account for the deviations from eqn (3) observed for their data in Fig. 3(a).

Air heat transfer correlation

Considering the thermophysical property differences between the experiments performed in air and liquids, and the larger LS/H dependence observed for some of the air results [1, 8], a separate correlation is proposed for the air data:

$$Nu_L = 0.239 Re_L^{0.655} Pr^{0.5} (\mu_i/\mu_h)^{0.13} \times (LS/H)^{0.18} [(H-B)/L]^{0.05} \quad (4)$$

which is valid for: $1267 < Re_L < 31\,250$; $Pr \approx 0.71$;

$0.025 < LS/H < 3.22$ and $0.06 < [(H-B)/L] < 3.0$. Predictions using equation (4) differ from the air data by an average error of $\pm 11.5\%$ (compared to $\pm 34\%$ with equation 3), and a standard deviation of 9.6%. This new correlation is plotted with the air data in Fig. 3(b). The scatter in the data may be a result of the low LS/H parameter values and high element-aspect ratios ($B/L = 1$) used by Rhee *et al.* [8]; also the elements in Anderson and Moffat [1] were not square ($L/L_z = 0.81$). In spite of these discrepancies, equation (4) represents an improvement over equation (3) in the prediction of air data by significantly reducing the average and maximum errors.

It is possible that the Prandtl number alone does not adequately characterize the scaling of convection heat transfer results between different fluids. Additional (or different) parameters may need to be used to represent the thermophysical-property effects, especially in the complex flows with large "roughness elements" considered here. Such efforts may lead to a more effective scaling across fluids, and could be undertaken as additional, well-characterized results become available for different fluids.

Comparison with existing correlations

The proposed correlations, equations (3) and (4), are now compared with existing correlations in the literature. Equation (3) is plotted with the correlation of Garimella and Schlitz [6, equation(3) in their paper] in Fig. 4(a) where the Nusselt numbers are normalized by $Pr^{0.5}(\mu/\mu_h)^{0.13}$. The data of Garimella and Schlitz [6] and Gudapati [9], in which $LS/H = 3.20$ and $(H-B)/L = 0.125$, are very well predicted by both correlations; however, the present correlation is valid over a much wider range of parameters.

In Fig. 4(b), the proposed correlation for the air data, equation (4), is compared with the data and correlations of Lehmann and Pembroke [4, equations (13a) and (13c)] and Wirtz and Dykshoorn [6, equation (7)]. For the first comparison, equation (4) is drawn for $LS/H = 0.096$ and $(H-B)/L = 1.24$ which represents the intermediate case of the three experimental configurations considered in [4]. In the upper Reynolds-number range of $15\,600 \leqslant Re_L \leqslant 31\,250$, the present correlation predicts the Lehmann–Pembroke data very well. For $1500 \leqslant Re_L \leqslant 12\,500$, however, the data are characterized by a different slope and deviate to a greater extent from equation (4). This change in slope was attributed to transition from laminar to turbulent flow [4]. In the same figure, equation (4) is also drawn at $LS/H = 2.0$ and $(H-B)/L = 0.25$, which lies in the middle of the five experimental configurations tested (see Table 1) by Wirtz and Dykshoorn [5]. The present correlation is essentially coincident with the Wirtz–Dykshoorn correlation.

CONCLUDING REMARKS

The data from the experiments listed in Table 1 were successfully correlated for a wide range of flow rates, fluids, and geometric configurations. The element length was found to be a good choice for the characteristic length in both

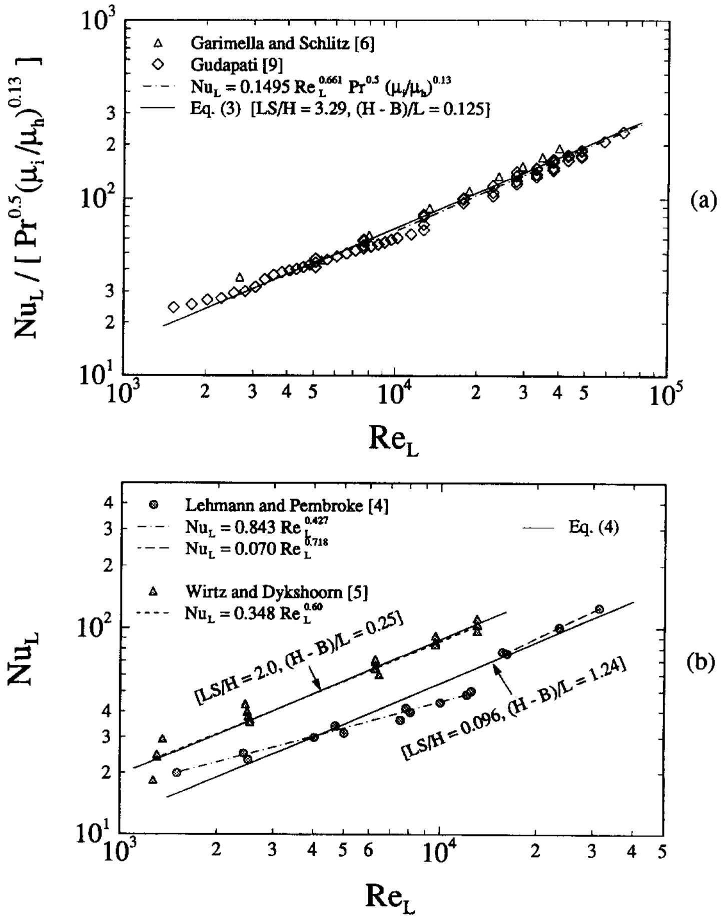

Fig. 4. Comparison of the proposed correlations, equations (3) and (4), with predictions and data in the literature, for liquids (a) and air (b).

the Reynolds and Nusselt numbers. The streamwise array spacing-to-channel height and clearance height-to-element length ratios, LS/H and $(H-B)/L$, were identified as important geometric parameters. A correlation for experiments performed in air, water and FC-77 was developed [equation (3)]. All the liquid data were very well correlated by the proposed correlation, but larger deviations with the air data prompted the development of an improved correlation for air data alone [equation (4)]. Both correlations compared favorably with predictive formulae in the literature.

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Heat transport along an oscillating flat plate in the presence of a transverse magnetic field

K. SHAILENDHRA and S. P. ANJALI DEVI
Department of Mathematics, Bharathiar University, Coimbatore 641 046, India

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1. INTRODUCTION

In recent years, several investigations were carried out to study the characteristics of a novel heat transport mode in which heat is transported from a hot to a cold reservoir by means of sinusoidal oscillations of a viscous fluid contained within open-ended tubes connecting the reservoirs. It has been experimentally [1] and analytically [2] confirmed that such periodic longitudinal oscillations result in a very significant enhancement in axial transport capability of the fluid.

A significant aspect of this thermal transport technique is that it involves no net convective mass transfer. The application of this method to several engineering problems is cited in ref. [3].

Very recently Kurzweg [4] showed that the same enhanced heat transfer process can occur in the classical Stokes problem of sinusoidally oscillating flat plate immersed within a viscous fluid of infinite extent when a constant temperature gradient is superimposed on the fluid parallel to the direction of oscillation of the plate.

However, so far no attempt has been made to study the effect of a magnetic field over such heat transport in electrically conducting fluid flows, despite the fact that in the devices using this thermal pumping, liquid metals such as mercury, liquid lithium or sodium are preferable [5]. Such an investigation will not only be useful in the design, control and improvement of liquid metal heat exchangers [6] but also may throw some light on the possible coupling of solar thermal system with liquid metal MHD heat exchangers [7].